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APPROXIMATE ANALYTICAL SOLUTIONS OF NONLINEAR HYPERBOLIC PARTIAL DIFFERENTIAL EQUATION

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Abstract: *The Multistep Modified Reduced Differential Transform Method (MMRDTM) is proposed and implemented in this study to obtain solutions of hyperbolic partial differential equations. We examine at the nonlinear Schrodinger equation (NLSE). Prior to implementing the multistep strategy, we switched the nonlinear term in the NLSE with the corresponding Adomian polynomials using the proposed technique. As a result, we can acquire solutions for the NLSE in a simpler and less difficult manner. Furthermore, the solutions can be estimated more precisely over a longer time period. We studied the NLS equation and graphed the features of this solution to show the strength and accurateness of the proposed technique.*

Keywords: *Adomian polynomials, multistep approach, Reduced Differential Transform Method, nonlinear Schrodinger equations*

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Introduction

Nonlinear Schrödinger equation equations occur in various fields in which nonlinear waves can be studied in fluid-filled viscoelastic tubes, solitary waves in semiconductors (thin plate), nonlinear optical waves, hydrodynamics and plasma waves. Coupled nonlinear Schrödinger equation is solved by using two approximate analytical methods such as Differential Transform Method (DTM) and Reduced Differential Transform Method (RDTM) by (Abazari, 2011). The numerical results demonstrate that the RDTM is very effective, convenient and quite accurate for nonlinear equation systems. Aruna and Ravi Kanth (2013) studied the approximate solutions of non-linear fractional Schrodinger equation by using two- dimensional DTM and Modified Differential Transform Method (MDTM). Three numerical tests were done to examine the efficacy and precision of the proposed methods. Rao (2016) dealt with RDTM to obtain the approximate solution of linear and NLS equations. This technique is better than numerical methods as it is error-free and does not involve big memory of the computer. It does not involve linearization, perturbation or discretization compared to other current techniques.

In the same year, Taghizadeh and Noori (2016) studied the NLS equation with cubic nonlinearity by using RDTM to obtain approximate solution. This approach can effectively be used for a wide range of problems. Li et al. (2017) considered a class of nonlinear Riesz space. The Multistep Modified Reduced Differential Transform Method (MMRDTM) is proposed and implemented in this study to discover solutions to hyperbolic partial differential equations. We'll examine at the nonlinear Schrodinger equation (NLSE) fractional Schrodinger equations. The semi-discrete and fully discrete systems are built on the basis of the conventional Galerkin finite element method in space and Crank-Nicolson difference method in time.

Inc and Korpinar (2017) introduced the residual power series method (RPSM) and homotopy analysis transform method (HATM) to obtain solution of Schrödinger equation of power law nonlinearity. The results demonstrate that in obtaining solution of the bright optical soliton of the NLS equation these methods are very effective and efficient. Hashemi and Akgül (2017) obtained analytical solution of NLS equation in both time and space fractional terms. Since analytical solutions are known for just a few cases, analyses of the properties of solutions are usually carried out numerically using such approaches. However, an analytical model that describes the dynamics of pulse propagation in a fiber is often desirable (Seadawy, 2012).

On the other hand, Ray (2013) suggested and implemented a modification to the fractional RDTM in order to solve fractional KdV equations. The modification in this strategy comprised the substitution of the nonlinear term by related Adomian polynomials. As a result, the solutions to the nonlinear problems can be achieved in a more straightforward manner with fewer computed terms. As a result, the solutions to the nonlinear problems can be achieved in a more straightforward manner with fewer computed terms. Furthermore, El-Zahar (2015) has presented an adaptive multistep DTM for solving singular perturbation initial-value problems. It generates the solution in a quick convergent series, resulting in the solution converging across a wide time range. It generates the solution in a quick convergent series, resulting in the solution converging across a wide time range.

Recently, Hussin et al. (2018) introduced and executed the Multistep Modified Reduced Differential Transform Method (MMRDTM) for solving nonlinear Schrodinger equations (NLSE). The outcomes demonstrate the approximate solutions of NLSE with high accuracy were obtained. Hussin et al. (2019a) also solved Klein-Gordon equations using MMRDTM and results showed that the MMRDTM is a valid and convenient method for finding analytic

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approximate solution of the Klein-Gordon equations. Besides that, Hussin et al. (2019b) obtained solution of fractional nonlinear Schrodinger equations (FNLSEs) by using MMRDTM.

In this study, we combine the modification made in Ray (2013) and the multistep approach in El-Zahar (2015) to execute a new technique called Multistep Modified Reduced Differential Transform Method (MMRDTM). The proposed method has the advantage of producing an analytical approximation in a fast-convergent sequence with fewer computed terms.

Methodology

The Development of Multistep Modified Reduced Differential Transform Method

Consider the general NLS equation of the form

$$iu_t + u_{xx} + \gamma|u|^2 u = 0, \qquad i = \sqrt{-1} \tag{1}$$

with initial condition

$$u(x,0) = f(x),$$

where γ is a constant and $u(x,t)$ is a complex function.

Using basic properties of MRDTM and then applying MRDTM to Equation (1), we can obtain

$$U_{k+1,m}(x) = \left(\frac{I}{k+1}\right)\left(\frac{\partial^2}{\partial x^2}\left(U_{k,m}(x)\right) + \gamma \sum_{k=0}^{n} A_{k,m}\right). \tag{2}$$

From the initial condition, write

$$U_0(x) = f(x).$$

Consider the general fractional nonlinear Korteweg-de Vries equation of the form (Ray, 2013)

$$u_t + (u^m)_x + (u^n)_{xxx} = 0 \qquad m > 0\,, 1 \le n \le 3\,, t > 0,\ \ 0 < \alpha \le 1, \tag{3}$$

where $K(m,n)$ denoted for the different values of m and n respectively. These $K(m,n)$ equations have the property that for certain values of m and n, their solitary wave solutions have compact support which is known as compactons (Rosenau, P. and Hyman, 1993).

Applying MMRDTM to Eq. (3) and using basic properties of MMRDTM, will obtain

$$U_{k+1,i}(x) = \left(\frac{1}{\mathrm{k+1}}\right)\left(-\frac{\partial}{\partial x}\left(A_{k,i}(x)\right) - \frac{\partial^3}{\partial x^3}\left(A_{k,i}(x)\right)\right) \tag{4}$$

with transformed initial condition

$$U_0(x) = f(x). \tag{5}$$

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Now, write the nonlinear term

$$N(u,t) = \sum_{n=0}^{\infty} A_n(U_0(x), U_1(x), \dots, U_n(x))t^n,$$

where A_n is the appropriate Adomian's polynomials. Recently, a novel method for calculating the Adomian polynomials was proposed by Kataria, K. K. and Vellaisamy (2016), namely

$$A_0 = N(U_0(x))$$

$$A_n(U_0(x), U_1(x), \dots, U_n(x)) = \frac{1}{2\pi}\int_{-\pi}^{\pi} N\left(\sum_{k=0}^{n} U_k(x)e^{ikx}\right)e^{-in\lambda}\, d\lambda, \quad n \geq 1$$

Replacing equation (5) into equation (4) and through iterative calculation, the $U_k(x)$ values can be obtained. Furthermore, the set of values $\{U_k(x)\}_{k=0}^{n}$ of the inverse transformation gives the n-terms approximate solution as follows

$$u(x,t) = \sum_{k=0}^{K} U_k(x)t^k, \qquad t \in [0,T].$$

Suppose that the interval $[0,T]$ is divided into M subintervals $[t_{i-1}, t_i], i = 1,2,\dots,M$, of equal step size $h = \frac{T}{M}$, by using the nodes $t_i = ih$. The key ideas of the MMRDTM are as follows. First, apply the modified RDTM over the interval $[0, t_1]$ to the initial value problem. From there, the approximate result

$$u_1(x,t) = \sum_{k=0}^{K} U_{k,1}(x)t^k, \qquad t \in [0, t_1]$$

is obtained by using the initial conditions $u(x,0) = f_0(x),\ u_1(x,0) = f_1(x)$. For $i \geq 2$, use the initial conditions $u_i(x, t_{i-1}) = u_{i-1}(x, t_{i-1}), (\partial/\partial t)u_i(x, t_{i-1})$ at each subinterval $[t_{i-1}, t_i]$, and the MRDTM is applied to the initial value problem over the interval $[t_{i-1}, t_i]$, where t_0 is replaced by t_{i-1}. Next multistep scheme for repeating process $u(x,0) = f_0(x), u_1(x,0) = a$.

The process is continued and repeated to create a sequence of approximate solutions $u_i(x,t), i = 1,2,\dots,M$, for the solution $u(x,t)$ such as

$$u_i(x,t) = \sum_{k=0}^{K} U_{k,i}(x)(t - t_{i-1})^k, \qquad t \in [t_{i-1}, t_i].$$

In fact, the MMRDTM assumes the following solution:

$$u(x,t) = \begin{cases} u_1(x,t) & t \in [0, t_1] \\ u_2(x,t) & t \in [t_1, t_2] \\ \vdots & \vdots \\ u_M(x,t) & t \in [t_{M-1}, t_M] \end{cases}.$$

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The new algorithm, MMRDTM, is simple in term of computational performance for all values of h. It is easily observed that if the step size $h = T$, then the MMRDTM reduces to the modified RDTM.

Finding

Example 1

Consider the one-dimensional NLS equation with zero trapping potential (Kanth and Aruna, 2009),

$$iu_t = -\frac{1}{2}u_{xx} - |u|^2 u \qquad ,t \geq 0 \tag{6}$$

subject to the initial condition

$$u(x,0) = e^{ix}.$$

The exact solution of this equation is $e^{i(x+\frac{t}{2})}$.

$$U_{k+1,m}(x) = \left(\frac{I}{k+1}\right)\left(\frac{1}{2}.\frac{\partial^2}{\partial x^2}(m(x)) + \sum_{k=0}^{n} A_{k,m}\right), \tag{7}$$

will be obtained by applying MRDTM to Equation (6) and using basic properties of MRDTM. From initial condition, write

$$U_0(x) = e^{ix}. \tag{8}$$

Replacing Equation (8) into Equation (7) and through iterative calculation, the $U_k(x)$ values can be obtained. Next, we list several set of values $\{U_6(x)\}_{k=0}^{6}$ of the inverse transformations gives the 6-terms approximate solution as follows,

$$u_1(x,t) = e^{Ix} + \frac{1}{2}\, Ie^{Ix}t - \frac{1}{8}\, e^{Ix}t^2 - \frac{1}{48}\, Ie^{Ix}t^3 + 0.002604166667\, e^{Ix}t^4$$

$$+0.000260416668\, Ie^{Ix}t^5 - 0.000021701388\, e^{Ix}t^6, \quad t \in [0,0.1].$$

$$\begin{aligned} u_2(x,t) = {} & 0.9987502604e^{Ix} + 0.04997916927\, Ie^{Ix} \\ & + (-0.02498958463 + 0.4993751301\, I)e^{Ix}(t-0.1) \\ & + (-0.1248437825 - 0.00624739615\, I)\, e^{Ix}(t-0.1)^2 \\ & + (0.001041232697 - 0.02080729713\, I)e^{Ix}(t-0.1)^3 \\ & + (0.002600912125 + 0.00013015409\, I)\, e^{Ix}(t-0.1)^4 \\ & + (-0.0000130154094 + 0.00026009121\, I)e^{Ix}(t-0.1)^5 \\ & + (-0.0000216742663 - 0.0000010846176\, I\,)e^{Ix}(t-0.1)^6, \quad t \\ & \in [0.1,0.2]. \end{aligned}$$

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$$u_3(x,t) = 0.9950041653e^{Ix} + 0.09983341664\, Ie^{Ix} + (-0.04991670837 + 0.4975020827\, I)e^{Ix}(t-0.2) + (-0.1243755206 - 0.01247917714\, I)\, e^{Ix}(t-0.2)^2 + (0.002079862866 - 0.02072925333\, I)e^{Ix}(t-0.2)^3 + (0.002591156672 + 0.0002599828592\, I)\, e^{Ix}(t-0.2)^4 + (-0.0000259982877 + 0.0002591156794\, I)e^{Ix}(t-0.2)^5 + (-0.00002159297977 - 0.000002166525309\, I\,)e^{Ix}(t-0.2)^6, \quad t \in [0.2, 0.3].$$

By using the nodes $t_m = mh$, divide the interval $[\,0,2\,]$ into 20 subintervals $[t_{m-1}, t_m], m = 1,2,\ldots,20$, equally sized with $h = 0.1$.

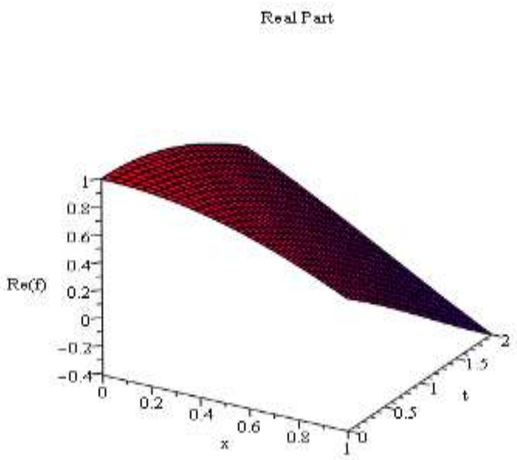

Figure 1 : Exact solution of Real Part

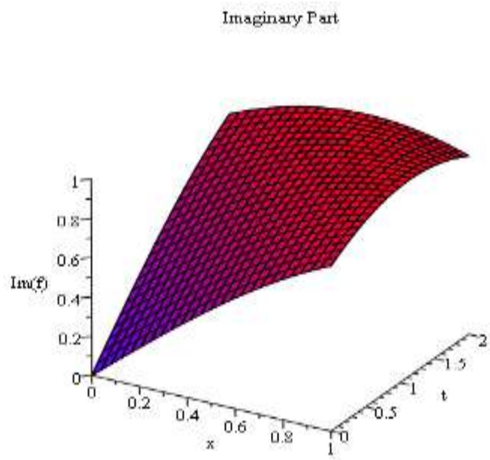

Figure 2 : Exact solution of Imaginary Part

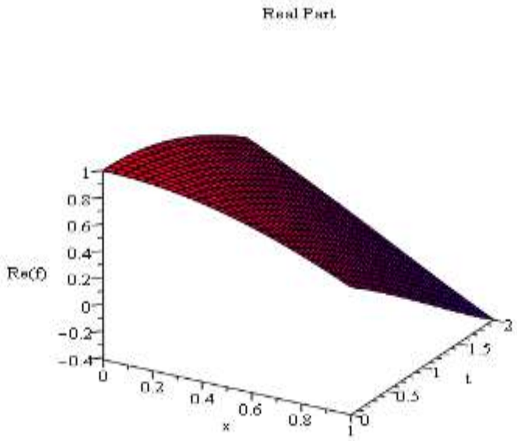

Figure 3 : MMRDTM of Real Part

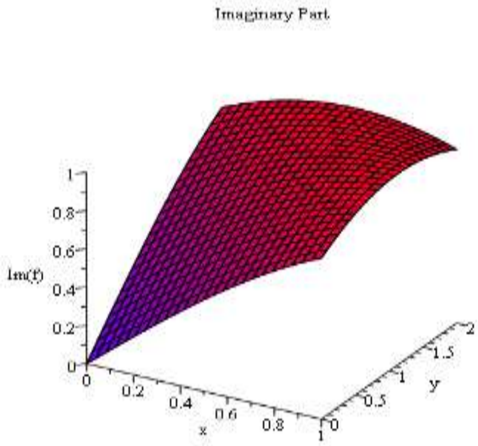

Figure 4 : MMRDTM of Imaginary Part

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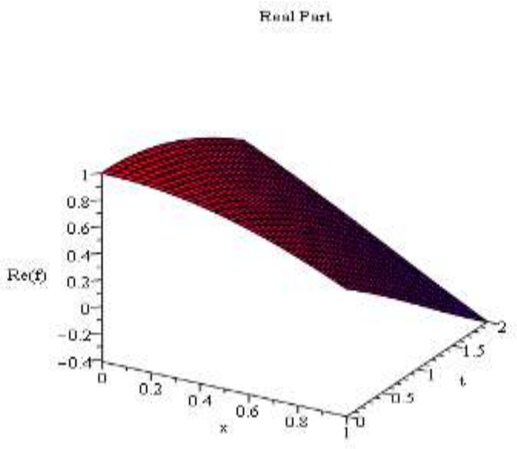

Figure 5 :MRDTM of Real Part

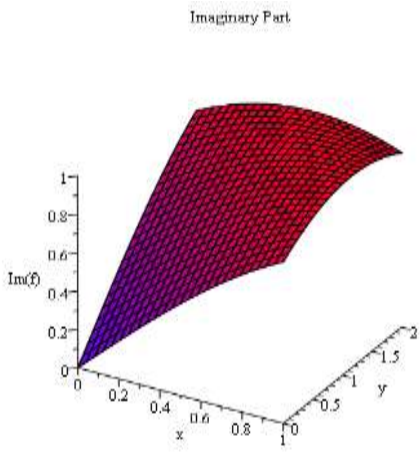

Figure 6 : MRDTM of Imaginary Part

Figure 1 and Figure 2 show the exact solutions, Figure 3 and Figure 4 show graphs of approximate solution MMRDTM while Figure 5 and Figure 6 show graphs of approximate solution MRDTM for $t \in [0,2]$ and $x \in [0,1]$ which involve real part and imaginary part. The shape of MMRDTM graphs look exactly similar to exact solutions. It is therefore obvious that the obtained solutions of MMRDTM for this form of NLS equation has minor error. The performance error analyses obtained by MMRDTM at $x = 1$ are summarized in Table 1 for real part. As we can see, the solution of MMRDTM is close to the exact solution.

Table 1 : Comparison solution of MMRDTM and MRDTM

t	Exact Solution	MMRDTM	MRDTM
0.1	$0.4975710479 + 0.8674232256 * I$	$0.4975710479 + 0.8674232256 * I$	$0.4975710479 + 0.8674232256 * I$
0.2	$0.4535961214 + 0.8912073601 * I$	$0.4535961215 + 0.8912073601 * I$	$0.4535961215 + 0.8912073600 * I$
0.3	$0.4084874409 + 0.9127639403 * I$	$0.4084874410 + 0.9127639403 * I$	$0.4084874407 + 0.9127639405 * I$
0.4	$0.3623577545 + 0.9320390860 * I$	$0.3623577546 + 0.9320390860 * I$	$0.3623577524 + 0.9320390873 * I$
0.5	$0.3153223624 + 0.9489846194 * I$	$0.3153223624 + 0.9489846195 * I$	$0.3153223520 + 0.9489846256 * I$
0.6	$0.2674988286 + 0.9635581854 * I$	$0.2674988286 + 0.9635581855 * I$	$0.2674987912 + 0.9635582079 * I$
0.7	$0.2190066871 + 0.9757233578 * I$	$0.2190066871 + 0.9757233579 * I$	$0.2190065768 + 0.9757234222 * I$
0.8	$0.1699671429 + 0.9854497300 * I$	$0.1699671428 + 0.9854497301 * I$	$0.1699668613 + 0.9854498915 * I$
0.9	$0.1205027694 + 0.9927129910 * I$	$0.1205027694 + 0.9927129911 * I$	$0.1205021247 + 0.9927133550 * I$
1.0	$0.07073720167 + 0.9974949866 * I$	$0.707372016e - 1 + 0.9974949867 * I$	$0.7073584964e - 1 + 0.9974957403 * I$
1.1	$0.02079482780 + 0.9997837642 * I$	$0.207948278e - 1 + 0.9997837643 * I$	$0.2079218486e - 1 + 0.9997852148 * I$
1.2	$-0.0291995223 + 0.9995736030 * I$	$-0.291995223e - 1 + 0.9995736031 * I$	$-0.2920439705e - 1 + 0.9995762399 * I$
1.3	$-0.07912088881 + 0.9968650285 * I$	$-0.791208887e - 1 + 0.9968650286 * I$	$-0.7912945048e - 1 + 0.9968695911 * I$

1.4	$-0.1288444943 + 0.9916648105 * I$	$-0.1288444944 + 0.9916648105 * I$	$-0.1288589193 + 0.9916723831 * I$
1.5	$-0.1782460556 + 0.9839859469 * I$	$-0.1782460556 + 0.9839859470 * I$	$-0.1782695018 + 0.9839980694 * I$
1.6	$-0.2272020947 + 0.9738476309 * I$	$-0.2272020948 + 0.9738476311 * I$	$-0.2272390315 + 0.9738664383 * I$
1.7	$-0.2755902468 + 0.9612752030 * I$	$-0.2755902470 + 0.9612752031 * I$	$-0.2756468585 + 0.9613035860 * I$
1.8	$-0.3232895669 + 0.9463000877 * I$	$-0.3232895671 + 0.9463000878 * I$	$-0.3233742479 + 0.9463418863 * I$
1.9	$-0.3701808314 + 0.9289597150 * I$	$-0.3701808317 + 0.9289597151 * I$	$-0.3703047767 + 0.9290199402 * I$
2.0	$-0.4161468365 + 0.9092974268 * I$	$-0.4161468370 + 0.9092974269 * I$	$-0.4163247500 + 0.9093825149 * I$

Conclusion

In this study, a new approximate analytical method known as MMRDTM is developed and implemented to obtain solution of one-dimensional nonlinear Schrodinger equations. The modification in this new strategy required replacing the nonlinear term with its Adomian polynomials and adopting a multistep approach. The results, as well as the graphical representations, demonstrated that the approximate NLSE solutions were achieved with great accuracy. The results as well as the graphical representations, demonstrated that the approximate NLSE solutions were achieved with great accuracy. Finally, we can state that the MMRDTM is reliable and efficient in providing analytic approximate solutions for this type of equation. Computation in this paper had been carried out by using Maple 13.

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